Study of $\bar{p}p \rightarrow \eta\pi^0\pi^0\pi^0$ at rest

The CRYSTAL BARREL Collaboration

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1 Abstract

Crystal Barrel data are presented on $\bar{p}p \to \eta\pi^0\pi^0\pi^0$ at rest in liquid hydrogen and also in gaseous hydrogen at 12 bar. Annihilation from the initial $^3P_0$ state is stronger in liquid than in gas by a factor 2.46 ± 0.15, in fair agreement with a prediction by Batty. There is a definite peak due to $\eta(1440)$. Liquid data determine its mass as $M = 1413$ MeV, $\Gamma = 49 \pm 8$ MeV. The mass is, however, lower in gas than in liquid by 12 ± 3 MeV; we attribute this mass shift to interference with broad background amplitudes. The $\eta(1440)$ decays dominantly to $\eta\sigma$: $\mathrm{BR}[a_0(980)\pi, a_0 \to \eta\pi]/\mathrm{BR}[\eta\sigma] = 0.4 \pm 0.2$. However, there is strong destructive interference between these two decay modes. There is

also a strong, broad $\eta\pi\pi$ component with $J^{PC} = 0^{-+}$, consistent with an earlier analysis proposing a very wide $\eta(1800)$ resonance; it contributes 31% of the $\eta\pi^0\pi^0\pi^0$ cross section in liquid. At the highest $\eta\pi\pi$ masses, there are definite 2^{-+} and 1^{++} signals, but we cannot establish precise resonance masses or widths. There is also evidence for the production of $f_2(1565)$, decaying to $a_2(1320)\pi$.

In order to study resonances in the $\eta\pi\pi$ channel, we have examined the reaction $\bar{p}p \to \eta\pi^0\pi^0\pi^0$ at rest. Resonances with $J^{PC} = 0^{-+}$, 1^{++} or 2^{-+} are produced only from initial P-states. Those with $J^{PC} = 2^{++}$ may be produced from the initial $^1S_0$ state. We present data from liquid hydrogen and also from hydrogen gas at 12 atmospheres, in order to study relative amounts of P-state annihilation in liquid and gas. Another essential objective is to study the $\eta\pi\pi$ resonance called $\eta(1440)$ by the Particle Data Group (PDG) [1]. We also find essential contributions from broad high mass $\eta\pi\pi$ states. Results have been presented in preliminary form at Hadron'95 and LEAP'96 conferences [2,3].

We begin with experimental details. The data were taken with the Crystal Barrel detector at LEAR, using antiproton beams of 300 MeV/c stopping in liquid hydrogen, or 200 MeV/c in hydrogen gas. A full technical description of the detector has been given earlier [4]. For present purposes, the γ detection is the essential element. A barrel of 1380 CsI crystals, each of 16 radiation lengths, covers 98% of the solid angle around the target, which is at the centre of the detector. Immediately surrounding the target are two multiwire chambers for liquid data, one for gas data; these chambers are used on-line to veto events containing charged particles. The resulting trigger selects a coincidence between the beam and final states containing only photon showers. A JET chamber, for detection of charged particles, surrounds the MWPCs. The last two layers are used in the on-line veto and remaining layers are used off-line as a further veto.

The CsI crystals have an energy resolution $\Delta E/E = 0.025E^{1/4}$, where E is photon energy in GeV; the angular resolution is ± 20 mrad in both polar and azimuthal angles. Events are discarded off-line when any energy deposit is centred in crystals immediately adjoining the entrance and exit beam-pipes;

this is to eliminate loss of shower energy into these holes. Consequently the acceptance is 95% of 4π, but the full 98% coverage is used to veto further photons.

Data reported here come from 8.2×10^6 triggers in liquid and 2.5×10^6 special triggers in gas. The data in gas used a trigger which selected events on-line with 7-11 separated showers in CsI crystals.

We now turn to analysis procedures. These are very close to those developed to study other neutral final states, and details are to be found in earlier papers [5,6]. The analysis chain selects 8γ final states and then pairs up photons to make $\eta\pi^0\pi^0\pi^0$ combinations. The final selection of events requires a confidence level $> 15\%$ for this final state. Potential backgrounds arise from $4\pi^0$, $\eta\eta$ with one $\eta \rightarrow 3\pi^0$, $\pi^0\eta'$ with $\eta' \rightarrow \eta\pi^0\pi^0$ and $\omega\omega\pi^0$, with both $\omega \rightarrow \pi^0\gamma$. These channels are rejected if they fit with confidence level $> 0.5\%$. The final sample consists of 5917 events in liquid and 5170 in gas.

A Monte Carlo simulation has been used to generate $\sim 48K$ $\eta\pi^0\pi^0\pi^0$ events in both liquid and gas satisfying criteria identical to those for data. They are used to evaluate the acceptance in the maximum likelihood fit described below. They determine a reconstruction efficiency of 16.7%. From this, we deduce a branching ratio in liquid of 1.8×10^{-4} of all annihilations.

The Monte Carlo study also investigates background levels from other channels masquerading as $\eta\pi^0\pi^0\pi^0$. The background comes almost entirely from $4\pi^0$ and is $(12 \pm 1.5)\%$ in liquid hydrogen and $(5 \pm 1)\%$ in gas; the difference originates from better detector performance on very soft photons for gas data, taken 4 years later than liquid data. The background follows a phase space distribution within experimental error. This phase space background is included in the amplitude analysis described below. We have also investigated the possibility that photons are incorrectly paired to π^0 or η; this effect is found to be below the 1% level and will be neglected.

We now consider general features of the data. Figs. 1 and 2 show projections on to $M(\pi^0\pi^0\pi^0)$, $M(\eta\pi^0\pi^0)$, $M(\pi^0\eta)$ and $M(\pi^0\pi^0)$ of data from liquid and gas. The obvious features are narrow peaks in $\eta\pi\pi$ at 1285 and 1410 MeV and $\eta\pi$ peaks due to $a_0(980)$ and $a_2(1320)$. From projections (a) and (b), it is evident that background channels $\eta\eta$ ($\eta \to 3\pi^0$) and $\pi\eta'$ ($\eta' \to \eta\pi^0\pi^0$) have been eliminated successfully. Histograms display the result of the maximum likelihood fit, which we now outline.

The $\eta\pi^0\pi^0\pi^0$ final state has $C = +1$, so the allowed initial states are $^1S_0$, $^3P_0$, $^3P_1$ and $^3P_2$. The channels we find to play a significant role are:

$$^3P_2 \to [a_2(1320)\sigma]_{\ell=0} \tag{1}$$
$$^3P_2 \to [a_0(980)\sigma]_{\ell=2} \tag{2}$$
$$^3P_0 \to [a_0(980)\sigma]_{\ell=0} \tag{3}$$
$$^3P_{2,1,0} \to [f_1(1285)\pi]_{\ell=1};\ f_1 \to [a_0(980)\pi]_{L=1} \tag{4}$$
$$^3P_2,^3P_0 \to [\eta(1440)\pi]_{\ell=2,0};\ \eta \to [a_0(980)\pi]_{L=0} \tag{5}$$
$$^3P_2,^3P_0 \to [\eta(1440)\pi]_{\ell=2,0};\ \eta \to [\eta\sigma]_{L=0} \tag{6}$$
$$^3P_2 \to [\eta_2(1645)\pi]_{\ell=0};\ \eta_2 \to [a_2(1320)\pi]_{L=0} \tag{7}$$
$$^3P_2,^3P_0 \to [\eta(1800)\pi]_{\ell=0};\ \eta(1800) \to [\eta\sigma]_{L=0} \tag{8}$$
$$^3P_{2,1,0} \to [f_1'(1700)\pi]_{\ell=1};\ f_1' \to [a_0(980)\pi]_{L=1} \tag{9}$$
$$^1S_0,^3P_1,^3P_2 \to [f_2(1565)\pi]_{\ell=2,1}; f_2(1565)\pi \to [a_2(1320)\pi]_{L=1}. \tag{10}$$

The only other channel one might anticipate is $^3P_0 \to \pi(1300)\eta$, but we find this to be negligible. The low branching ratio of 1.8×10^{-4} is consistent with mostly P-state annihilation.

In reactions (1)–(10), ℓ is the orbital angular momentum of the final state in the production process; L is the angular momentum of a resonance decay. The σ is a shorthand for the $\pi\pi$ S-wave amplitude, which is parametrised accurately over the required mass range [7]. The $\eta_2(1645)$ is the 2^{-+} $I = 0$ resonance we reported earlier in a study of $\eta\pi^0\pi^0\pi^0$ in flight [8]. The $\eta(1800)$ is a very broad 0^- resonance visible in J/Ψ radiative decays to $\eta\pi\pi$ and other channels [9]. We find it plays an essential role here. We also find a large and unavoidable contribution from a 1^{++} state f_1' with a mass in the range 1600–1800 MeV.

When we began this analysis, we were apprehensive that the combinatorics might make it difficult to identify and separate broad components described by channels (7)–(10). What we have found is that the combinatorics indeed spoil the determination of masses and widths of broad resonances. Without these broad resonances, the high mass range of $\eta\pi\pi$ is not fitted accurately, so something is definitely required there. However, individually, reactions (7),(9) and (10) cannot be separated by looking at mass projections. They are so close to the top of the available $\eta\pi\pi$ mass range that fits are unstable against masses drifting upwards and increasing in width. We find, however, that the individual processes are well separated by their angular dependence and we are confident of the requirement for all of the four broad components in channels (7)–(10). For each channel there are specific Clebsch-Gordan coefficients and angular dependence for each step of a decay, e.g. $\eta_2(1645) \to a_2(1320) \to \eta\pi$. We have tried scrambling these Clebsch-Gordan coefficients and angular dependences. Wrong expressions reduce fitted signals to a noise level close to that expected statistically. However, the correct expressions make each channel leap into view with large improvements in log likelihood. It is unlikely that faults in the detector or Monte Carlo could simulate the complicated angular dependence of these signals. In order to err on the safe side, we keep only channels which contribute at least a 6σ effect, or whose presence is required logically. The hardest problem is to separate $^3P_2$, $^3P_1$ and $^3P_0$ contributions to 1^+ $\eta\pi\pi$ final states, as discussed below for $f_1(1285)$.

The data have been fitted using two independent programmes, one of which uses relativistic tensors, and the second uses Wick rotations as outlined in ref. [8]. These two descriptions differ slightly by relativistic effects built into the tensor expressions. In practice, such differences are small, and the two programmes cross-check one another accurately. The amplitude f for channel (4), as an example, then takes the form:

$$f = g\ BW(f_1)F(a_0)\exp(-\alpha p^2)B_1(p)B_1(p')Z. \tag{11}$$

Here g is a complex coupling constant. Then BW is a relativistic Breit-Wigner amplitude of constant width for the f_1 resonance, and F is the Flatté form for $a_0(980)$ [10]. The $\eta(1800)$ is described using the s-dependent width shown graphically in ref. [9]. The Zemach amplitude Z describes angular dependence and B is a Blatt-Weisskopf form factor with a radius of 0.8 fm; results are insensitive to the precise radius. The momentum in the production process is p, and the decay momentum of the f_1 to $a_0\pi$ (in this example) is p'. The exponential is a form factor which reproduces closely the Vandermeulen form factor [11] with $\alpha = 1.5$ GeV^{-2} and accounts for the well known observation that low momentum (high mass) final states are favoured. Results are however insensitive to α. All combinations of π^0 in each of reactions (1)–(10) are included coherently. Cross sections from $^3P_2$, $^3P_1$, $^3P_0$ and $^1S_0$ are added incoherently.

The likelihood function, L, is defined in the standard way [8] so that a one standard deviation change in one variable affects $ln\ L$ by 0.5. Gas and liquid data are fitted simultaneously (and separately to test their consistency).

In discussing the physics, we shall refer to Table 1, which shows (i) percentage contributions of each process to the final fit, (ii) changes $\Delta(ln\ L)$ when each component is dropped one by one and remaining contributions are refitted. Our past experience is that, with these statistics, a change in $ln\ L$ of 40 can be considered decisive (statistically $> 8\sigma$, but subject to some systematic error). For some processes, eg. ${}^3P_1 \to f_1(1285)$, $\Delta(ln\ L)$ is below this level, but it is logical to keep this contribution because of the obvious presence of ${}^3P_2$ and ${}^3P_0 \to f_1(1285)$.

We now return to the gross features of Figs. 1 and 2, considering first the relative rates of P-state annihilation in liquid and gas. Batty has predicted [12] an enhancement of ${}^3P_0$ annihilation in liquid (L) compared to gas (G):

$$r({}^3P_0) = \frac{\sigma({}^3P_0)_L/\sigma({}^3P_0)_G}{\sigma({}^3P_2)_L/\sigma({}^3P_2)_G} \simeq 1.7. \qquad (12)$$

The origin of this result is that the ${}^3P_0$ level has a width due to annihilation which is larger than ${}^3P_2$ or ${}^3P_1$ states [13]. Stark mixing in the atomic cascade leads to stronger population of the ${}^3P_0$ level, and this effect is stronger in liquid than in gas. It turns out that this factor can be determined rather precisely from a simultaneous analysis of our data in liquid and gas.

From Figs. 1 and 2, the $a_2(1320)$ signal is a factor 2 smaller in liquid than in gas. It comes entirely from the ${}^3P_2$ initial state, either via the $a_2\sigma$ reaction or via ${}^3P_2 \to \eta_2(1645)\pi$, followed by decay of the resonance to $a_2\pi$. What is actually happening is that the background is increased in liquid because of production of $\eta(1800)$ from ${}^3P_0$. This immediately indicates an enhancement of ${}^3P_0$ in liquid of at least a factor 2. Likewise, the $f_1(1285)$ signal goes down in liquid by a factor 1.6 compared to gas. This again points towards ${}^3P_0$ enhancement in liquid. The amplitude analysis finds an enhancement factor $r({}^3P_0) = 2.46 \pm 0.15$, where the error covers statistics and also systematic variations from all of a large number of fits using varying ingredients. In this determination, the broad $\eta(1800)$ plays a strong role; varying its mass from 1.6 to 2.2 GeV, $r(^3P_0)$ varies from 2.39 to 2.51. Without it in the fit, the mass projections in liquid cannot be reproduced accurately. An example is shown in Fig. 3.

Batty predicts that $^3P_1$ annihilation compared with $^3P_2$ is a factor 0.77 weaker in liquid than in gas. The data do not determine this factor with any accuracy. The reason is that $^3P_1$ annihilation is rather small and is not well determined. We set this factor to 0.77, though conclusions change very little if it is set to 1.0. In fitting production of $f_2(1565)$, we shall assume that 50% of annihilation is from P-states in gas and 10% in liquid.

Next we consider the narrow peaks in $\eta\pi\pi$, starting with the 1285 MeV peak. Its mass optimises at 1283.8 MeV, consistent within experimental errors with the PDG value of 1282.2 MeV [1]. We fit with $\Gamma = 27$ MeV, obtained by folding the experimental resolution of 10 MeV into the Lorentz line shape. Let θ be the angle in the rest frame of $f_1(1285)$ between its production direction and its decay to $a_0(980)$. Then $^3P_2$ annihilation leads to an angular distribution $(3 + \cos^2\theta)$, $^3P_1 \rightarrow \sin^2\theta$ and $^3P_0 \rightarrow \cos^2\theta$. The data demand a strong θ dependence: $1 + (0.34 \pm 0.05)\cos^2\theta$ in gas, $1 + (0.98 \pm 0.05)\cos^2\theta$ in liquid, so it is certain that a large 1^+ component due to $f_1(1285)$ is present. The larger $\cos^2\theta$ component in liquid again indicates larger $^3P_0$ annihilation than in gas.

However, relative amounts of $^3P_2$, $^3P_1$ and $^3P_0$ annihilation are not easy to determine. Table 1 shows that dropping all annihilation to $f_1(1285)$ has a bigger effect on log likelihood than the sum of changes when individual contributions are dropped. So there is obviously a degree of ambiguity in the way annihilation is to be attributed to $^3P_2$, $^3P_1$ and $^3P_0$. This ambiguity is partially resolved by interferences with other components. The ambiguity makes it difficult to estimate the possible presence of $\eta(1295)$, whose isotropic angular dependence can be simulated by a suitable mix of P-state production of $f_1(1285)$. Because the mass and width of $\eta(1295)$ are rather higher than $f_1(1285)$ [1], we are able to place an upper limit on its contribution of 10% of $f_1(1285)$ with 95% confidence. However, if its mass and width were really degenerate with $f_1(1285)$, its contribution could rise to 30%.

Next we consider the 1415 MeV peak in $\eta\pi\pi$. This is explained naturally by $\eta(1440)$. There is marginal evidence for contamination by $f_1(1420)$. However, we shall show that properties of $\eta(1440)$ are affected little by this possible contamination.

Data from liquid determine a mass of 1415 ± 2 MeV. Data from gas fit to a mass of 1401 ± 2 MeV. The difference in mass is clearly visible by eye in Fig. 4, which shows the peaks on an expanded scale. It is not due to experimental error, since the peaks due to $f_1(1285)$ optimise within 1 MeV of one another. Our experience with the Crystal Barrel detector from observations of other narrow resonances is that the mass scale has an error not worse than ± 2 MeV.

The explanation of the mass shift seems to be interference with the broad $a_2(1320)\sigma$ signal and the $\eta(1800)$ and possibly with other broad contributions. If background and $\eta(1440)$ are in phase, the peak agrees with the mass of $\eta(1440)$. However, when they differ in phase, interferences between real parts of the amplitudes alter the shape and position of the peak. We have found by varying the broad components that interferences are easily capable of moving the peaks in either or both of liquid and gas by up to 15 MeV. Uncertainty about the precise form of the broad backgrounds therefore makes the mass of $\eta(1440)$ uncertain by this amount. In a separate fit to liquid data or gas data alone, the phase of the narrow peak can be adjusted to reproduce the peak in an optimum way. However, in a combined fit, one should demand consistency between gas data and liquid data for $^3P_2$ and $^3P_0 \to \eta(1440)$ in both $\eta\sigma$ and $a_0\pi$ decays. It proves to be difficult to adjust the background to achieve a perfect match in phase to place the peaks at precisely the right separation between liquid and gas. The reason is that the background phase cannot be varied freely because of large interferences over all of phase space; these have a statistical weight bigger than the small $\eta(1440)$ peaks. The fitted peak is very sensitive to the mass, so we have eventually allowed the peak to take slightly different mass in liquid and gas, but insist on the same width. The fitted width, $\Gamma = 49 \pm 9$ MeV is less than in previous analyses because of coherence with the background; without this interference, the width would increase to ~ 70 MeV, consistent with the PDG value.

A second curious feature is that production of $\eta(1440)$ is stronger from $^3P_2$ with $\ell = 2$ than from $^3P_0$ with $\ell = 0$. This is required independently by data in liquid and in gas. To some extent it is due to the multiplicity factor $(2J + 1)$, which favours $^3P_2$; but one would expect the centrifugal barrier to suppress the $\ell = 2$ cross section by a factor 4. The strong production from $^3P_2$ is therefore a surprise. If $^3P_2$ production of $\eta(1440)$ is omitted, the fit is unable to reproduce the full height of the $\eta(1440)$ peak in gas. Decays of the resonance are isotropic for one $\eta\pi\pi$ combination. Therefore the distinction between $^3P_2$ and $^3P_0$ annihilation arises only from interferences. It is possible that the broad signals we are fitting to channels (7)–(10) are somehow biasing the fit towards $^3P_2$ production of $\eta(1440)$, but we have been unable to locate a possible origin for such bias. If $^3P_2$ and $^3P_0$ annihilation are constrained to the same strength in gas, log likelihood rises by 15, but otherwise conclusions remain unchanged.

A further complication is the possible presence of $f_1(1420)$. This is believed to decay largely to $K\bar{K}\pi$ rather than $\eta\pi\pi$. However, other data on $\bar{p}p \rightarrow \pi^0(K\bar{K}\pi)$ [14] can only set an upper limit on the possible contribution of $f_1(1420)$ to present data of a few %, comparable with the magnitude of the 1415 MeV peak. We have therefore repeated the fit adding the possibility of ${}^3P_2$, ${}^3P_1$ and ${}^3P_0 \rightarrow f_1(1420)\pi$, followed by f_1 decays to $\eta\sigma$ or $a_0(980)\pi$; the $f_1(1420)$ mass and width are fixed at PDG values. Log likelihood improves by 18.3 for the addition of 8 parameters. Statistically this is a 4.3σ effect, but systematic uncertainties in the $\eta(1440)$ line shape dilute this significance to some extent. Fortunately the presence of $f_1(1420)$ has rather small effects on the parameters fitted to $\eta(1440)$.

We now consider the branching ratio of $\eta(1440)$ to $a_0(980)\pi$ and $\eta\sigma$. The point requiring careful attention is that these decays overlap on the Dalitz plot; Fig. 5 shows the data within a window of $\eta\pi\pi$ mass from 1365 to 1445 MeV. We find a strong destructive interference between $a_0(980)\pi$ and $\eta\sigma$ having a big effect on branching ratios.

Cross sections for all processes involving $\eta(1440)$ and the possible $f_1(1420)$ are collected into Table 2. Columns 2 and 3 show that $\eta(1440)$ decays dominantly to $\eta\sigma$; column 4 shows strong destructive interference between the two decay modes. If $f_1(1420)$ is added into the analysis (entries labelled (b)), destructive interference is again presents, but more $a_0\pi$ is fitted to $f_1(1420)$. The angular distribution fitted to $f_1(1420)$ is fortuitously the same in liquid and gas: $1 - 0.24\cos^2\theta$; however, the $\cos^2\theta$ term is barely significant. So it is possible that what is being fitted as $f_1(1420)$ is really due to $\eta(1440)$ and some small failure to reproduce its line shape. Remember that we use a Breit-Wigner amplitude of constant width, whereas in reality $K\bar{K}\pi$ and $\rho\rho$ channels are opening in this vicinity. Lack of information on branching ratios presently prevents a reliable improvement on the constant width approximation.

Separate analyses of data from liquid and gas agree closely on $\eta\sigma$ and $a_0(980)\pi$ contributions shown in Table 2. So do independent determinations of the branching ratio from $^3P_2$ and $^3P_0$ annihilation. From the fit omitting $f_1(1420)$, the ratio of $a_0(980)\pi$ and $\eta\sigma$ branching ratios is

$$r_{1440} = \frac{\sigma(a_0\pi)}{\sigma(\eta\sigma)} = 0.32, \tag{13}$$

and the interference term compared to the overall branching ratio is

$$r_{int} = \frac{\sigma(interference)}{\sigma(total)} = -1.18. \tag{14}$$

With $f_1(1420)$ included in the analysis, $r_{1440} \rightarrow 0.37$ for the $\eta(1440)$ alone and $r_{int} \rightarrow -1.27$; if the $f_1(1420)$ contribution is lumped in with $\eta(1440)$, $r_{1440} \rightarrow 0.52$ and $r_{int} \rightarrow -0.85$. A compromise which covers all the fits we have made is

$$r_{1440} = 0.4 \pm 0.2, \tag{15}$$
$$r_{int} = -1.2 \pm 0.4, \tag{16}$$

where the errors cover systematic uncertainties. The result for r_{1440} differs from one of our own earlier publications [15]. Those data were statistically superior, but suffered from a large $\omega a_0(980)$ background which had to be removed in the analysis. Our present result agrees reasonably well with the result of the GAMS group shown at the LEAP'96 conference [16], namely $r_{1440} = 0.24 \pm 0.05$.

We have made a second independent analysis of the $\eta\sigma$ and $a_0(980)\pi$ branching ratios of $\eta(1440)$ using quite a different technique. This alternative approach is the Matched Filter technique, familiar in applications to electronics where weak signals are to be isolated from noise [17]. The essential idea is to fit the $\eta\pi\pi$ and $\eta\pi$ mass projections with a background varying slowly with $M(\eta\pi\pi)$ plus a narrow peak due to (a) $\eta(1440) \rightarrow \eta\sigma$ and (b) $\eta(1440) \rightarrow a_0(980)\pi$. This ignores interference with the broad background and with reflections of $\eta(1440)$. Resulting equations take the form:

$$\sigma_a(M^2) = Ar_{aa}(M^2) + Br_{ab}(M^2) + C(M^2) \tag{17}$$
$$\sigma_b(M^2) = Ar_{ba}(M^2) + Br_{bb}(M^2) + D(M^2). \tag{18}$$

Here $\sigma_{a,b}$ are data multiplied by cross sections for processes a,b, but varying the mass M of the fitted $\eta(1440)$ across the whole mass range of the experimental peak. The terms r_{aa}, r_{bb} are autocorrelation functions of the signals, and r_{ab} is the cross-correlation function. Terms C and D are smooth backgrounds, to be fitted empirically. The auto-correlation functions r describe the narrow peak in the data and coefficents A and B measure the strengths of cross sections for the two processes. The result of this approach is $r_{1440} = 0.3 \pm 0.15$, in good agreement with the amplitude analysis.

Finally we discuss contributions from high mass $\eta\pi\pi$ resonances. We find it unavoidable to include a large broad component due to ${}^3P_0$ annihilation to $\eta(1800)$. This component approximates to phase space, but is somewhat peaked towards high $\eta\pi$ masses. It far exceeds the magnitude of possible experimental background. It is largely responsible for reducing the magnitude of the $a_2(1320)$ peak in liquid. In Ref. (9), where $\eta(1440)$ was observed in $J/\Psi \rightarrow \gamma(\eta\pi\pi)$, their Fig. 3(a) showed that the peak height of the narrow $\eta(1440)$ was slightly smaller than the broad $\eta(1800)$ on which it sits; we find a very similar result here after dropping combinatorics, as shown on Fig. 6. We find no requirement for $\eta(1800) \rightarrow a_0(980)\pi$, in accord with ref [9]. However, such a component might be confused with the weak $a_0(980)\sigma$ channel.

The magnitude of that channel sets an upper limit on $\eta(1800) \to a_0(980)\pi$ of $\sim 20\%$ of $\eta(1800) \to \eta\sigma$. The production of the broad $\eta(1800)$ from $^3P_2$ with $\ell = 2$ is weak because it is cut off at high masses by the centrifugal barrier for production. The phase space background we include to allow for contamination from $4\pi^0$ has no visible effect on any plots, and its effect is solely to reduce the contribution of $\eta(1800)$ by $\sim 20\%$ of its magnitude, i.e. from 38% to 31% in liquid.

We find that $\eta_2(1645)$ plays an indispensible role in fitting the data. Without it, $ln\ L$ is worse by 152.9, a decisive amount. We have fixed the mass and width from our earlier work. If the decay to $a_0\pi$ with $L = 2$ is added to the analysis, $ln\ L$ improves by 16.1, and the branching ratio is $\sim 20\%$ of $a_2\pi$. If the $\eta\sigma$ decay with $L = 2$ is added instead, $ln\ L$ improves by 24.3 and the branching ratio is $\sim 10\%$ of $a_2\pi$. However, since we cannot identify a clear peak in the data due to $\eta_2(1645)$, we do not regard either decay mode as established definitively. The $a_0\pi$ and $\eta\sigma$ decays were not observed in our earlier work [8].

To our surprise, there is an even stronger requirement for the presence of a 1^{++} signal decaying to $a_0(980)\pi$ with $L = 1$. We find no significant evidence for it decaying to $\eta\sigma$ with $L = 1$ or $a_2(1320)\pi$ with $L = 1$. A radial excitation of $f_1(1285)$ is to be expected roughly in the mass range 1600–1700 MeV. Lee et al [18] have observed a candidate for its $I = 1$ partner at 1700 MeV. The mass difference between $I = 1$ and $I = 0$ states is likely to be small, so we fix the mass of the 1^{++} signal at 1700 MeV and its width at 400 MeV. The quality of fits are insensitive to precise parameters of 1^+ and 2^- states.

In an earlier paper [19], we have presented evidence for a 2^+ resonance at 1540 MeV, decaying weakly to $\pi\pi$ and more strongly to $\omega\omega$. This is the only resonance which can be produced from the initial $^1S_0$ state. However, it is inhibitied by an $\ell = 2$ centrifugal barrier in the production reaction, and by an $L = 1$ centrifugal barrier in its decay to $a_2(1320)\pi$, hence $\eta\pi^0\pi^0$. Adding it to the fit, $ln\ L$ improves by 73.3 for the addition of 5 extra parameters. Results are insensitive to its width, which we take as 150 MeV, but there is a weak optimum at a mass of 1580 MeV. The small branching fraction of $f_2(1565)$ in Table 1 presumably reflects the suppression by the centrifugal barriers for production and decay, and the small phase space available to $a_2(1320)\pi$ decays.

Finally we report two negative results. We have scanned the mass range 1450-1600 MeV for $f_1(1510) \rightarrow \eta\pi\pi$ using $\Gamma = 50$ MeV. We find no evidence for the presence of this resonance. Secondly, we have inspected plots of the form of Figs 1 and 2 for 50 MeV slices of $\eta\pi\pi$ mass, but observe no effects requiring physics beyond channels (1) - (10).

In summary, the main points to emerge from these data are as follows:

- (a) $^3P_0$ annihilation is enhanced in liquid compared to gas by a factor 2.46 ± 0.15,

- (b) $\eta(1440)$ is observed with $M \simeq 1410$ MeV, $\Gamma = 49 \pm 9$ MeV, and decays dominantly to $\eta\sigma$, but with destructive interference with the $a_0(980)\pi$ decay mode; there is a definite difference in the peak mass in liquid and gas, suggesting interference with a background amplitude and leading to an uncertainty in mass of about 15 MeV;

- (c) the presence of $\eta_2(1645)$ decaying to $a_2(1320)\pi$ is required;

- (d) there is evidence for a strong 1^{++} contribution in $\eta\pi\pi$ in the mass range above 1600 MeV;

– (e) there is some evidence for the presence of $f_2(1565) \to a_2(1320)\pi$.

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Figure Captions

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Figure 1 Projections of data from gas on to (a) $M(\pi^0\pi^0\pi^0)$, (b) $M(\eta\pi^0\pi^0)$, (c) $M(\pi^0\eta)$ and (d) $M(\pi^0\pi^0)$. Histograms show the maximum likelihood fit.

Figure 2 Projections of data from liquid on to (a) $M(\pi^0\pi^0\pi^0)$, (b) $M(\eta\pi^0\pi^0)$, (c) $M(\pi^0\eta)$ and (d) $M(\pi^0\pi^0)$. Histograms show the maximum likelihood fit.

Figure 3 Projections of data from liquid on to $M(\pi^0\eta)$. The histogram shows the maximum likelihood fit when $\eta(1800)$ is omitted.

Figure 4 The $\eta\pi\pi$ mass projection around 1285 and 1410 MeV peaks from gas (full line) and liquid (dashed).

Figure 5 Dalitz plot for $\eta\pi\pi$ masses of 1365 to 1445 MeV in gas; $M^2(\pi^0\pi^0)$ is plotted vertically in GeV^2 and $M^2(\eta\pi^0)$ horizontally in GeV^2.

Figure 6 Data (crosses) compared with the 0^- cross section projected on to $M(\eta\pi\pi)$ from one $\eta\pi^0\pi^0$ combination only, and normalised to the $\eta(1440)$ peak.

Channel	% gas	% liquid	$\Delta(ln\ L)$
${}^3P_2 \to a_2\sigma$	65.2	44.8	1361.6
${}^3P_2 \to [a_0\sigma]_{L=2}$	5.2	3.5	71.0
${}^3P_0 \to a_0\sigma$	3.9	6.4	49.2
${}^3P_2 \to f_1(1285)$	11.5	7.5	104.8
${}^3P_1 \to f_1(1285)$	2.6	1.7	6.1
${}^3P_0 \to f_1(1285)$	2.4	3.9	69.6
${}^3P_2 \to [\eta(1440)]_{\ell=2} \to a_0\pi$	1.5	0.9	23.6
${}^3P_2 \to [\eta(1440)]_{\ell=2} \to \eta\sigma$	4.8	3.2	23.6
${}^3P_0 \to [\eta(1440)]_{\ell=0} \to a_0\pi$	0.2	0.8	15.6
${}^3P_0 \to [\eta(1440)]_{\ell=0} \to \eta\sigma$	0.4	1.4	10.1
${}^3P_2 \to \eta_2(1645) \to a_2\pi$	13.2	9.1	152.9
${}^3P_0 \to \eta(1800)$	18.4	30.6	167.8
${}^3P_2 \to \eta(1800)$	3.7	2.6	11.3
${}^3P_2 \to f_1(1700)$	2.6	1.7	21.6
${}^3P_1 \to f_1(1700)$	4.2	2.9	23.6
${}^3P_0 \to f_1(1700)$	6.8	11.1	139.4
${}^3P_2 \to f_2(1565)$	3.2	2.2	67.3
${}^3P_1 \to f_2(1565)$	2.4	1.3	3.3
${}^1S_0 \to f_2(1565)$	0.4	2.3	5.9
All $f_1(1285)$	16.7	12.9	253.5
All $\eta(1440)$	3.3	2.8	59.3
All $f_2(1540)$	6.3	5.7	73.3
All $f_1(1700)$	13.7	15.1	247.7
All $\eta(1800)$	22.1	33.2	179.1

Table 1
Percentage contributions from each process, keeeping interferences within a channel but ignoring interferences with other channels; change in lnL when each component is dropped and others are refitted. Percentage contributions do not sum to 100% because of interferences between channels.

		$\sigma(a_0\pi)$	$\sigma(\eta\sigma)$	Interference	Total	r_{1440}	r_{int}
(a)	$^3P_2$ gas	1.5	4.8	-3.5	2.8	0.32	-1.18
	$^3P_0$	0.2	0.8	-0.5	0.5		
	$^3P_2$ liquid	0.9	3.2	-2.1	1.9		
	$^3P_0$	0.4	1.4	-0.9	0.9		
(b)	$^3P_2$ gas $\rightarrow \eta(1440)$	2.0	5.1	-4.1	3.1	0.37	-1.27
	$^3P_0$	0.2	0.4	-0.3	0.2		
	$^3P_2$ liquid $\rightarrow \eta(1440)$	1.3	3.6	-2.7	2.2		
	$^3P_0$	0.2	0.7	-0.5	0.4		
	$^3P_2$ gas $\rightarrow f_1(1420)$	1.3	1.3	-0.8	1.9	0.89	-0.39
	$^3P_1$	0.7	0.9	-0.5	1.2		
	$^3P_0$	0.1	0.1	-0.0	0.1		
	$^3P_2$ liquid $\rightarrow f_1(1420)$	0.9	1.0	-0.5	1.35		
	$^3P_1$	0.5	0.7	-0.3	0.8		
	$^3P_0$	0.1	0.1	-0.1	0.1		

Table 2
Cross sections (as a percentage of all $\bar{p}p \rightarrow \eta 3\pi^0$) for (a) $\eta(1440) \rightarrow a_0(980)\pi$ and $\eta\sigma$, and the interference between them, (b) adding possible $f_1(1420)$ to the analysis.